

Introduction to algebraic topology : Week 10

Exercise 1. Let C_\bullet be a chain complex.

- 1) Define the notion of a subcomplex $B_\bullet \subset C_\bullet$.
- 2) If B_\bullet is a subcomplex of C_\bullet , define the quotient complex C_\bullet/B_\bullet .
- 3) In the situation of 2), show that $\partial_{k+1}(C_{k+1}) + B_k \subset \partial_k^{-1}(B_{k-1})$ and that $H_k(C_\bullet/B_\bullet)$ can be identified with $\partial_k^{-1}(B_{k-1})/(\partial_{k+1}(C_{k+1}) + B_k)$.

Exercise 2. Let X, A be topological spaces and $r : X \rightarrow A$ be a retract. Show that for all integer $n \geq 0$ we have an isomorphism of homology groups

$$H_n(X) \simeq \ker(r_*) \oplus H_n(A).$$