Introduction to algebraic topology : Week 11

Exercise 1. Prove the ZigZag lemma : If

$$0 \longrightarrow \mathrm{C}_\bullet \longrightarrow \mathrm{D}_\bullet \longrightarrow \mathrm{E}_\bullet \longrightarrow 0$$

is a short exact sequence of chain complexes then for all integer $n > 0$ there exists a homomorphism

$$\delta_n : H_n(\mathrm{E}_\bullet) \longrightarrow H_{n-1}(\mathrm{C}_\bullet)$$

called the connecting homomorphism such that the sequence

$$\dots \xrightarrow{\delta_{n+1}} H_n(\mathrm{C}_\bullet) \xrightarrow{F_*} H_n(\mathrm{D}_\bullet) \xrightarrow{G_*} H_n(\mathrm{E}_\bullet) \xrightarrow{\delta_n} H_{n-1}(\mathrm{C}_\bullet) \xrightarrow{F_*} \dots$$

is exact.

Exercise 2. Compute the homology groups of the torus $\mathbb{T}^2$.

Exercise 3. Let (X_1, x_1),...,(X_n, x_n) be pointed manifolds. Use the points $x_1, ...x_n$ to form the wedge product $X_1 \vee \cdots \vee X_n$. Show that for all $p > 0$, $H_p(X_1 \vee \cdots \vee X_n) = H_p(X_1) \oplus \cdots \oplus H_p(X_n)$.