Introduction to algebraic topology : Week 13

Exercise 1. Let $f : \mathbb{S}^n \longrightarrow \mathbb{S}^n$ be a continuous map. Show that if $\deg(f) \neq (-1)^{n+1}$ then f has a fixed point.

Exercise 2.

1) Show that the antipodal map $\alpha : \mathbb{S}^n \longrightarrow \mathbb{S}^n$ is homotopic to the identity if and only if n is odd.
2) Show that there exists a nowhere vanishing vector field (ie section of the tangent bundle, identified with a map $V : \mathbb{S}^n \longrightarrow \mathbb{R}^{n+1}$ such that $x \perp V(x)$ or all $x \in \mathbb{S}^{n+1}$) on $\mathbb{S}^n$ if and only if n is odd.

This is the Hairy Ball Theorem. See Lee th.13.32.