

## Introduction to algebraic topology : Week 6

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**Exercise 1.** Consider the action of  $\mathbb{Z}$  on  $\mathbb{R}^m \setminus \{0\}$  given by  $n.x = 2^n x$ .

- (a) Show that the quotient map for this action is a covering map
- (b) Show that the quotient space  $\mathbb{R}^m \setminus \{0\}/\mathbb{Z}$  is homeomorphic to  $\mathbb{S}^{m-1} \times \mathbb{S}^1$ .
- (c) If  $m \geq 3$ , find the universal covering of  $\mathbb{S}^{m-1} \times \mathbb{S}^1$ .

**Exercise 2.** For a positive integer  $n$ , let  $C_n$  be the circle of radius  $\frac{1}{2^n}$  centered at the point  $(\frac{1}{2^n}, 0)$ . Let  $C$  be the union of all these circles (it is sometimes called a clamshell).

- (a) Show that  $C$  is connected and locally path-connected, but not semilocally simply connected.
- (b) Identify  $C$  with  $C \times \{0\} \subset \mathbb{R}^3$  and let  $X$  be the cone over  $C$ , that is the union of all segments from points in  $C$  to the point  $(0, 0, 1)$ .

Show that  $X$  is semilocally simply connected but not locally simply connected.