# Introduction to algebraic topology : Week 8

---

**Exercise 1.** Let $G$ be a group and $Y \longrightarrow X = Y/G$ be a $G$-covering with $Y$ connected. If $T$ and $T'$ are $G$-sets, show that any covering morphism $Y_T \longrightarrow Y_{T'}$ comes from a map of $G$-sets from $T$ to $T'$. *(Recall that $Y_T$ is the quotient of $Y \times T$ by the diagonal action of $G$, same for $T'$.)*

If $T \longrightarrow T'$ and $T' \longrightarrow T''$ are maps of $G$-sets inducing maps $Y_T \longrightarrow Y_{T'}$ and $Y_{T'} \longrightarrow Y_{T''}$ show that the composition $Y_T \longrightarrow Y_{T''}$ is induced by the composition $T \longrightarrow T''$.

**Exercise 2.** Let $G$ be a connected and locally path connected topological group with neutral element $e$ and $p : E \longrightarrow G$ be a covering map with $E$ connected. Show that $E$ can be made into a topological group such that $p$ is a group homomorphism.

Suppose furthermore that $G$ has a universal covering $u : \tilde{G} \longrightarrow G$. Show that $\ker(u)$ and $\pi_1(G, e)$ are isomorphic as groups.

**Exercise 3.** *Base change.*

Let $f : (Z, z) \longrightarrow (X, x)$ be a map of connected, locally path connected spaces that have universal covering spaces.

1) Describe a natural functor F : Cov(X) $\longrightarrow$ Cov(Z) from coverings of $X$ to coverings of $Z$. (Prove that your construction is well-defined and functorial)
2) Describe a natural functor G : $\pi_1(X, x) - \text{sets} \longrightarrow \pi_1(Z, z) - \text{sets}$. (Prove that your construction is well-defined and functorial)
3) Show that these constructions commute with fiber functors, that is we have a commutative diagram of categories

$$\begin{array}{ccc} Cov(X) & \xrightarrow{F} & Cov(Z) \\ {\scriptstyle Fib_x}\downarrow & & \downarrow{\scriptstyle Fib_z} \\ \pi_1(X, x) - \text{sets} & \xrightarrow{G} & \pi_1(Z, z) - \text{sets} \end{array}$$